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Modulus Dependence on Large Scale Porosity of Powder Metallurgy Steel

P.G. Allison, M.F. Horstemeyer, and H.R. Brown

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This article compares the existing theoretical expressions for the porosity dependence on elastic constants to experimental data for a commercially available material, FC-0205 powder metallurgy (PM) steel. The modulus of compression, tension, effective torsion, and ultrasound-based data at varying porosity levels are plotted graphically against the theoretical expressions. An equation by McAdam (*J. Iron Steel Inst. Lond.*, 1950, 168, p 346) was able to most accurately predict the experimental data with the adjustment of only one material constant.

Keywords carbon/alloy steels, mechanical testing, powder metallurgy

1. Background and discussion

The use of porous solids for a variety of engineering applications has led the researchers to perform the experimental and theoretical studies of material-processed effects of the pores on the elastic constants (Ref 1-11). The earliest study of the effective bulk and shear moduli calculations by Mackenzie (Ref 2) were conducted when developing a theory for sintering. The pores were assumed to be homogeneous in size and spherical in shape, and were embedded in a sphere of dense material. His calculation was only valid when a linear relationship between stress and strain was present. The relationships derived for the shear and bulk moduli are as follows:

$$1-\frac{G}{G_0}=\frac{5(3K_0+G_0)}{9K_0+G_0}\varphi+X\varphi^2 \qquad \text{(Eq 1)}$$

$$\frac{1}{K}=\frac{1}{K_0(1-\varphi)}+\frac{3\varphi}{4G_0(1-\varphi)}+X\varphi^2 \qquad \text{(Eq 2)}$$

where G is the shear modulus, K is the bulk modulus, φ is the porosity (void or pore volume fraction), and the subscript 0 refers to the zero porosity (fully dense) material. The coefficient X was not determined by Mackenzie, resulting in his formulas being exact only when the porosity was much less than unity, and so the modulus is then linear with porosity.

Experimental study by McAdam (Ref 1) on sintered ferrous alloys allowed for the determination of the elastic modulus from static bending tests. The elasticity of the alloys decreased with an increasing porosity, whereas a more prominent decrease was observed when the porosity was higher than 20%. A theoretical expression for the effective elastic modulus relating to porosity was determined by curve fitting the experimental data giving the following expression:

$$\frac{E}{E_0}=(1-\varphi)^x \qquad \text{(Eq 3)}$$

where E/E_0 is the ratio of the aggregate elastic Young's modulus divided by the dense elastic modulus, and x is a material-specific constant.

An approach by Hashin (Ref 3) and by Hashin and Shtrikman (Ref 4) consisted of finding upper and lower bounds for the elastic moduli. The bounds were determined by incorporating the change in strain energy of a homogenous matrix caused by the addition of nonhomogeneities. The research concluded that, for many practical applications, accurate predictions of shear modulus bounds were determined except when the nonhomogeneities were pores or rigid inclusions.

Later independent studies by both Hill (Ref 5) and Budiansky (Ref 6) developed a self-consistent theory that focused on a void imbedded in the matrix of a simple geometrical configuration with unknown effective moduli. An implicit equation of moduli was then obtained by determining the average stress and strain distributions in the matrix and void, in the presence of an externally applied stress or strain.

Buch and Goldschimdt (Ref 7) extended Hashin's model to examine the porosity effects on the elastic moduli for sintered metals. Experimental and theoretical results were compared for sintered iron, copper, and tungsten. The authors assumed that the expression for the upper bound of the shear modulus developed by Hashin was equivalent to Young's modulus if the Poisson's ratio was constant over a certain range yielding the expression:

$$\frac{E}{E_0}=1-\frac{15(1-\nu_0)\varphi}{7-5\nu_0+2(4-5\nu_0)\varphi} \qquad \text{(Eq 4)}$$

where ν is the Poisson's ratio. This simplified expression provided fairly accurate predictions when there was a constant sintering temperature and a high relative density material.

P.G. Allison, Geotechnical & Structures Laboratory, Engineer Research & Development Center, US Army Corps of Engineers, 3909 Halls Ferry Road, Vicksburg, MS 39180; and Department of Mechanical Engineering, Mississippi State University, PO Box ME, Mississippi State, MS 39762, USA and **M.F. Horstemeyer** and **H.R. Brown**, Department of Mechanical Engineering, Mississippi State University, PO Box ME, Mississippi State, MS 39762. Contact e-mail: paul.g.allison@usace.army.mil.

A stress concentration factor was incorporated in the expression by Bert (Ref 8) that predicted normalized Young's modulus with porosity as depicted by

$$\frac{E}{E_0} = (1 - \varphi)^{K_0 V_{max}} \quad \text{(Eq 5)}$$

where K_0 is an isolated pore's stress concentration factor, and V_{max} is the maximum possible pore length for a nonequiaxed pore. Values of K_0 and V_{max} were given in tabular form for different pore geometries and loading directions. This expression was more accurate for spherical pores embedded in a matrix when the porosity value was less than 20%. As porosity was increased, Eq 5 predicted lower values than were experimentally determined.

Lemaitre and Dufailly (Ref 9) combined the laws of elasticity and damage for the one dimensional case as given by

$$\varepsilon_e = \frac{\sigma}{E_0(1 - \varphi^{2/3})} = \frac{\sigma}{E} \quad \text{(Eq 6)}$$

where ε_e is the elastic strain; and σ is the stress. For pure ductile damage with no residual microstress, the modulus was expressed in terms of porosity by

$$\frac{E}{E_0} = \left(1 - \varphi^{2/3}\right) \quad \text{(Eq 7)}$$

Experimental study by Spitzig et al. (Ref 10) on sintered iron compacts showed that the Young's and shear moduli compared well to those of the theoretical studies by Hill (Ref 5) and Budiansky (Ref 6). Although a close correlation of the theoretical studies to the data was observed, the data were linear because porosity levels up to 12% were tested. The researchers also noticed that the expression

$$\frac{E}{E_0} = \frac{G}{G_0} = (1 - 2\varphi) \quad \text{(Eq 8)}$$

accurately predicted the relationship of the experimental data.

Ramakrishnan and Arunachalam (Ref 11) modified the study by Hashin and Shtrikman (Ref 4) using a spherical pore surrounded by a spherical matrix with an increasing pressure on the pore surface caused by the interaction of multiple pores in relation the modulus to the porosity. The resulting expression for normalized Young's modulus was

$$\frac{E}{E_0} = \frac{(1 - \varphi)^2}{1 + (2 - 3\nu_0)\varphi} \quad \text{(Eq 9)}$$

The theoretical results matched porous ceramics experimental data except in the case when the porosity level increased to 40%, and the Poisson's ratio was around 0.3.

Porous-sintered steel experimental data for Young's modulus as a function of porosity and those of the theoretical equation of Ramakrishnan and Arunachalam (Ref 11) were compared by Chawla and Deng (Ref 12). A close correlation was noted with a linear relationship to the maximum porosity of 10%.

A constitutive model developed by Bammann et al. (Ref 13) for predicting failure in ductile materials uses the expression

$$\frac{E}{E_0} = (1 - \varphi) \quad \text{(Eq 10)}$$

to capture the effect that voids have on the degradation of the elastic moduli.

A study of porosity effects on the elastic response of cast steels was detailed by Hardin and Beckermann (Ref 14). The empirical relationship for elastic modulus by Ref 8 was compared to experimental data at varying porosities when the elastic modulus was found to decrease nonlinearly with increasing porosity. For cast steels, Hardin and Beckermann (Ref 14) found the following relationship:

$$\frac{E}{E_0} = (1 - 2\varphi)^{2.5} \quad \text{(Eq 11)}$$

Hardin and Beckermann (Ref 14) also discovered that besides the amount of porosity, factors, such as porosity distribution, pore shape, and pore size, dictate the stiffness of porous materials.

A review by Haynes (Ref 15) examined multiple theoretical analyses on the effect that porosity exerts on the elastic response of sintered porous metals that had been presented up to the time of the publication. Experimental data for steel from McAdam (Ref 1) were plotted against theoretical results showing that the data from Ref 2-5, 4, and 16 predicted linear and nonlinear modulus-dependent porosities with higher Young's modulus values than those experimentally determined when the porosity level was less than 20%. Haynes noted that the best agreement between experimental and theoretical results was achieved by Hashin and Shtrikman (Ref 4) whose expression satisfactorily captured the material response at low porosities. Hashin and Shtrikman attributed the lack of correlation to experimental data at higher porosity levels to irregular-shaped and lower density necks between sintered particles that were assumed to have lower elastic modulus than the fully dense material. However, Haynes has reported that no evidence has been provided that sintered necks are physically different than those of the bulk solid, and suggested that differences between experiments and theory are caused by the stress concentrations of voids.

The aforementioned theoretical results are compared to experimental data as shown in Fig. 1. Pores were assumed spherical and randomly distributed in the matrix for the theories with the Young's modulus derived from the bulk and shear moduli when no expression for Young's modulus was supplied. Normalized Young's modulus experimental results plotted are from compression, tension, and ultrasound experiments for FC-0205 PM steel that have been detailed in an earlier study (Ref 17). The average and standard deviation of the sintered blank specimens are provided in Table 1.

The experimental data and the theoretical predictions are plotted in Fig. 1 depicting the elastic modulus dependence related to porosity. When plotting the tensile elastic modulus as a function of porosity for FC-0205 steel, the models of Mackenzie, McAdam, Hashin, Budiansky, Hill, Buch, Spitzig, and Ramakrishnan all give reasonable predictions when accounting for the uncertainty within the experimental data. The lower-bound predictions were from Ramakrishnan, while the upper-bound predictions were from Lemaitre. Spitzig was not only able to predict the tensile data, but also the ultrasound-based data at 19% porosity. Bert and Bammann were able to predict the 10% porosity tensile data and the ultrasound-based data at 12 and 14% porosities. However, the data at higher porosities were overestimated by Bert and Bammann. Hardin captured the 10% porosity tensile data and 12% porosity ultrasound-based data, but at higher porosity levels, the modulus was overpredicted. McAdam's equation was able to

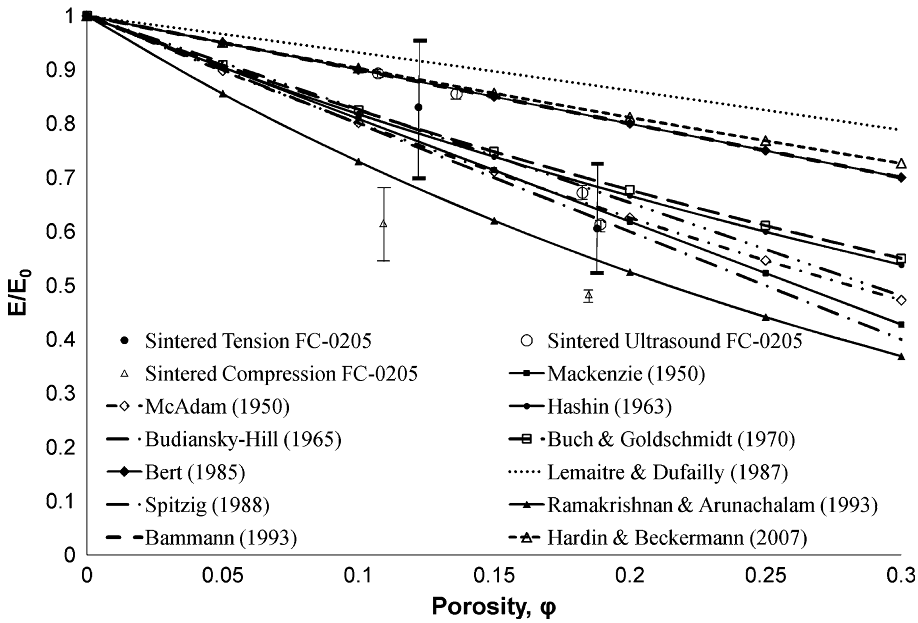

Fig. 1 Young's modulus dependence on porosity comparison of FC-0205 experimental data to theoretical predictions

Table 1 Density and porosity distribution of FC-0205 sintered blanks used for modulus determination (average and standard deviation)

Ultrasound-based		Tension		Compression	
Density, g/cm^3	Porosity, %	Density, g/cm^3	Porosity, %	Density, g/cm^3	Porosity, %
6.31 ± 0.01	18.9 ± 1.0	6.32 ± 0.01	18.8 ± 0.00	6.34 ± 0.02	18.5 ± 0.00
6.36 ± 0.02	18.3 ± 1.0	…	…	…	…
6.72 ± 0.03	13.6 ± 1.0	…	…	…	…
6.94 ± 0.05	10.7 ± 1.0	6.83 ± 0.02	12.2 ± 0.00	6.93 ± 0.00	10.9 ± 0.00

most accurately predict the experimental data by adjusting the material constant to capture the tensile data and the higher porosity ultrasound-based data.

2. Conclusions

This article compared FC-0205 experimental tension, compression, and ultrasound-based data to theoretical expressions for porosity dependence on elastic constants. From the comparisons, it was found that certain theoretical expressions performed better than others at predicting the experimental data. However, a theoretical expression by McAdam (Ref 1), which only has one material constant, appeared to most accurately capture the experimental data over the range of porosity levels.

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